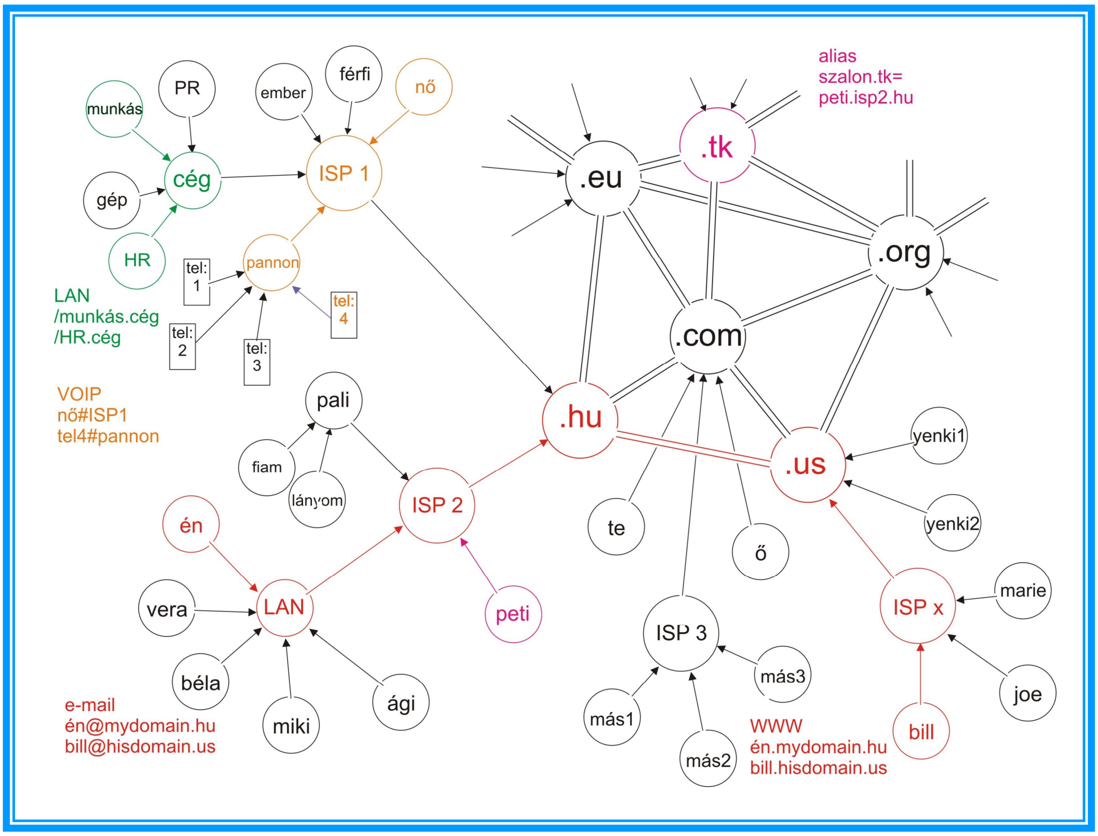
# Some words about Networks.

## II. part


Compiled by

Dr. Peter G. Gyarmati


for research purposes.





TCC COMPUTER STUDIO

2010.

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Compiled by Dr. Peter G. Gyarmati

for research purposes,

based on Wikipedia pages,

2010. 03. 6.

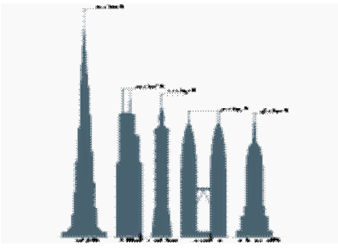
# 1. Cluster diagram

A **Cluster diagram** or *clustering diagram* is a general type of diagram, which represents some kind of cluster. A cluster in general is a group or bunch of several discrete items that are close to each other. [1]

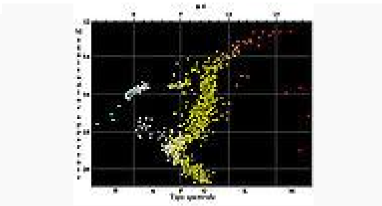
The cluster diagram figures a cluster, such as a network diagram figures a network, a flow diagram a process or movement of objects, and a tree diagram an abstract tree. But all these diagrams can be considered interconnected: A network diagram can be seen as a special orderly arranged kind of cluster diagram. A cluster diagram is a mesh kind of network diagram. A flow diagram can be seen as a line type of network diagram, and a tree diagram a tree type of network diagram.

## Types of cluster diagrams

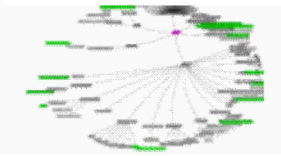
Specific types of cluster diagrams are:



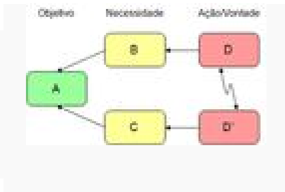
Comparison of sky scraper



Astronomic cluster of the Messier 3 globular cluster

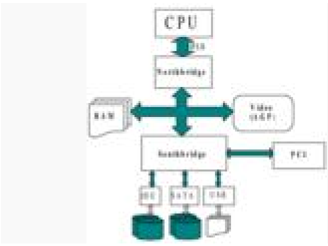


Biositemap

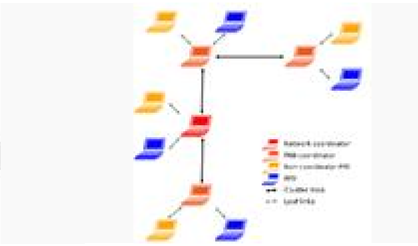



Cluster chart in brainstorming

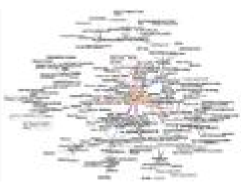
- In architecture a comparison diagram is sometimes called a cluster diagram.[2]
- In astronomy diagrams of star cluster, galaxy groups and clusters or globular cluster.
- In brainstorming a cluster diagrams is also called *cloud diagram*. They can be considered "are a type of non-linear graphic organizer that can help to systematize the generation of ideas based upon a central topic. Using this type of diagram... can more easily brainstorm a theme, associate about an idea, or explore a new subject".[3] Also, the term cluster diagrams is sometimes used as synonym of mind maps".[4]



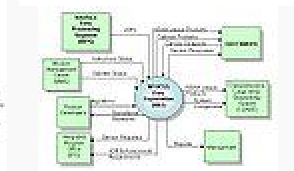
Computer architecture of a PC



Computer network

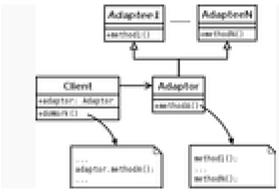


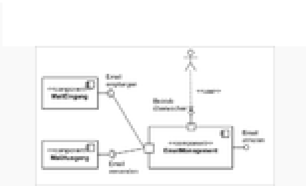
Internet

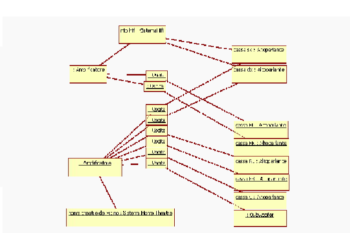


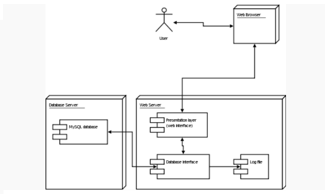
System context

- In computer science more complex diagrams of computer networks, computer architecture, file systems and internet can be considered cluster diagrams.
- In information visualization specific visual representation of large-scale collections of non-numerical information are sometimes drawn in the shape of a cluster diagram.
- In quantum field theory for example, according to Crawford (1998), the called *coupled cluster diagram* is a "simple diagrammatic formalism popularized by Kucharski and Bartlett [in 1986] by which one may construct the coupled cluster energy and amplitude equations far more quickly than by direct application of Wick's theorem".[5]


UML Class


UML Component


UML Composite structure


UML Deployment

- In the Unified Modeling Language (UML) all structure diagrams can be considered cluster diagrams. These structure diagrams emphasize what things must be in the system being modeled. UML encounters here the Class diagram, Component diagram, Composite structure diagram, Deployment diagram, Object diagram, and the Package diagram.

## References

1. ^ Lemma cluster at Wiktionary. Retrieved Sept 18, 2008.
2. ^ Illustration called City of London Skyscraper Cluster Diagram at skyscrapernews.com. Retrieved 18 september 2008. Comment: This illustration depicts a "comparison diagram", but yet is called a "cluster diagram".
3. ^ Cluster/Cloud Diagrams at enchantedlearning.com. 2003-2009. Accessed Nov 17, 2009.
4. ^ Cluster diagrams are another way to mind map by starting with the keywords first. at www.brainstorming-that-works.com. Retrieved 18 september 2008.
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## Further reading

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## External links

- Cluster diagram at cap.nsw.edu.au
- City of London Skyscraper Cluster Diagram

# 2. Scale-free network

A **scale-free network** is a network whose degree distribution follows a power law, at least asymptotically. That is, the fraction $P(k)$ of nodes in the network having $k$ connections to other nodes goes for large values of $k$ as $P(k) \sim k^{-\gamma}$ where $\gamma$ is a constant whose value is typically in the range $2<\gamma<3$, although occasionally it may lie outside these bounds.

Scale-free networks are noteworthy because many empirically observed networks appear to be scale-free, including the protein networks, citation networks, and some social networks.[1]

## Highlights

- Scale-free networks show a power law degree distribution like many real networks.
- The mechanism of preferential attachment has been proposed as an underlying generative model to explain power law degree distributions in some networks.
- It has also been demonstrated [2] that scale-free topologies in networks of *fixed* sizes can arise as a result of Dual Phase Evolution.

## History

In studies of the networks of citations between scientific papers, Derek de Solla Price showed in 1965 that the number of links to papers—i.e., the number of citations they receive—had a heavy-tailed distribution following a

Pareto distribution or power law, and thus that the citation network was scale-free. He did not however use the term "scale-free network" (which was not coined until some decades later). In a later paper in 1976, Price also proposed a mechanism to explain the occurrence of power laws in citation networks, which he called "cumulative advantage" but which is today more commonly known under the name preferential attachment.

Recent interest in scale-free networks started in 1999 with work by Albert-László Barabási and colleagues at the University of Notre Dame who mapped the topology of a portion of the Web (Barabási and Albert 1999), finding that some nodes, which they called "hubs", had many more connections than others and that the network as a whole had a power-law distribution of the number of links connecting to a node.

After finding that a few other networks, including some social and biological networks, also had heavy-tailed degree distributions, Barabási and collaborators coined the term "scale-free network" to describe the class of networks that exhibit a power-law degree distribution. Soon after, Amaral et al. showed that most of the real-world networks can be classified into two large categories according to the decay of P(k) for large k. Caroline S. Wagner (2008) demonstrated that scientific collaboration at the global level falls into scale free network structures along a power law form.

Barabási and Albert proposed a mechanism to explain the appearance of the power-law distribution, which they called "preferential attachment" and which is essentially the same as that proposed by Price. Analytic solutions for this mechanism (also similar to the solution of Price) were presented in 2000 by Dorogovtsev, Mendes and Samukhin and independently by Krapivsky, Redner, and Leyvraz, and later rigorously proved by mathematician Béla Bollobás. Notably, however, this mechanism only produces a specific subset of networks in the scale-free class, and many alternative mechanisms have been discovered since.

Although the scientific community is still debating the usefulness of the scale-free term in reference to networks, Li et al. (2005) recently offered a potentially more precise "scale-free metric". Briefly, let *g* be a graph with edge-set ε, and let the degree (number of edges) at a vertex *i* be $d_i$. Define

$$s(g) = \sum_{(i,j)\in\epsilon} d_i d_j.$$

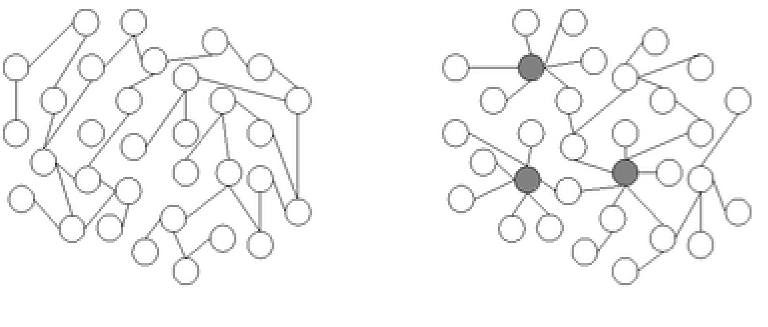
This is maximised when high-degree nodes are connected to other high-degree nodes. Now define

$$S(g) = \frac{s(g)}{s_{max}}$$

where $s_{max}$ is the maximum value of *s*(*h*) for *h* in the set of all graphs with an identical degree distribution to *g*. This gives a metric between 0 and 1, such that graphs with low *S*(*g*) are "scale-rich", and graphs with *S*(*g*) close to 1 are "scale-free". This definition captures the notion of self-similarity implied in the name "scale-free".

## Characteristics and examples

Random network (a) and scale-free network (b). In the scale-free network, the larger hubs are highlighted.



(a) Random network (b) Scale-free network

As with all systems characterized by a power law distribution, the most notable characteristic in a scale-free network is the relative commonness of vertices with a degree that greatly exceeds the average. The highest-degree nodes are often called "hubs", and are thought to serve specific purposes in their networks, although this depends greatly on the domain.

The power law distribution highly influences the network topology. It turns out that the major hubs are closely followed by smaller ones. These, in turn, are followed by other nodes with an even smaller degree and so on. This hierarchy allows for fault tolerant behaviour in the face of random failures: since the vast majority of nodes are those with small degree, the likelihood that a hub would be affected is almost negligible. Even if such event occurs, the network will not lose its connectedness, which is guaranteed by the remaining hubs. On the other hand, if a few major hubs are removed from the network, it simply falls apart and is turned into a set of rather isolated graphs. Thus hubs are both the strength of scale-free networks and their Achilles' heel.

Another important characteristic of scale-free networks is the clustering coefficient distribution, which decreases as the node degree increases. This distribution also follows a power law. That means that the low-degree nodes belong to very dense sub-graphs and those sub-graphs are connected to each other through hubs. Consider a social network in which nodes are people and links are acquaintance relationships between people. It is easy to see that people tend to form communities, i.e., small groups in which everyone knows everyone (one can think of such community as a complete graph). In addition, the members of a community also have a few acquaintance relationships to people outside that community. Some people, however, are so related to other people (e.g., celebrities, politicians) that they are connected to a large number of communities. Those people may be considered the hubs responsible for making such networks small-world networks.

At present, the more specific characteristics of scale-free networks can only be discussed in either the context of the generative mechanism used to create them, or the context of a particular real-world network thought to be scale-free. For instance, networks generated by preferential attachment typically place the high-degree vertices in the middle of the network, connecting them together to form a core, with progressively lower-degree nodes making up the regions between the core and the periphery. Many interesting results are known for this subclass of scale-free networks. For instance, the random removal of even a large fraction of vertices impacts the overall connectedness of the network very little, while targeted attacks destroys the connectedness very quickly. Other scale-free networks, which place the high-degree vertices at the periphery, do not exhibit these properties; notably, the structure of the Internet is more like this latter kind of network than the kind built by preferential attachment. Indeed, many of the results about scale-free networks have been claimed to apply to the Internet, but are disputed by Internet researchers and engineers.[*citation needed*]

As with most disordered networks, such as the small world network model, the average distance between two vertices in the network is very small relative to a highly ordered network such as a lattice. The clustering coefficient of scale-free networks can vary significantly depending on other topological details, and there are now generative mechanisms that allow one to create such networks that have a high density of triangles.

It is interesting that Cohen and Havlin proved that uncorrelated power-law graphs having $2 < \gamma < 3$ will also have ultrasmall diameter $d \sim \ln \ln N$. So from the practical point of view, the diameter of a growing scale-free network might be considered almost constant.

Although many real-world networks are thought to be scale-free, the evidence remains inconclusive, primarily because the generative mechanisms proposed have not been rigorously validated against the real-world data. As such, it is too early to rule out alternative hypotheses. A few examples of networks claimed to be scale-free include:

- Some social networks, including collaboration networks. An example that has been studied extensively is the collaboration of movie actors in films.
- Protein-Protein interaction networks.
- Networks of sexual partners in humans, which affects the dispersal of sexually transmitted diseases.
- Many kinds of computer networks, including the World Wide Web.
- Semantic networks. [3]

## Generative models

These scale-free networks do not arise by chance alone. Erdős and Rényi (1960) studied a model of growth for graphs in which, at each step, two nodes are chosen uniformly at random and a link is inserted between them. The properties of these random graphs are not consistent with the properties observed in scale-free networks, and therefore a model for this growth process is needed.

The scale-free properties of the Web have been studied, and its distribution of links is very close to a power law, because there are a few Web sites with huge numbers of links, which benefit from a good placement in search engines and an established presence on the Web. Those sites are the ones that attract more of the new links. This has been called the winner takes all phenomenon.

The most widely known generative model for a subset of scale-free networks is Barabási and Albert's (1999) rich get richer generative model in which each new Web page creates links to existing Web pages with a probability distribution which is not uniform, but proportional to the current in-degree of Web pages. This model was originally discovered by Derek J. de Solla Price in 1965 under the term **cumulative advantage**, but did not reach popularity until Barabási rediscovered the results under its current name (BA Model). According to this process, a page with many in-links will attract more in-links than a regular page. This generates a power-law but the resulting graph differs from the actual Web graph in other properties such as the presence of small tightly connected communities. More general models and networks characteristics have been proposed and studied (for a review see the book by Dorogovtsev and Mendes).

A different generative model is the **copy** model studied by Kumar et al. (2000), in which new nodes choose an existent node at random and copy a fraction of the links of the existent node. This also generates a power law.

However, if we look at communities of interests in a specific topic, discarding the major hubs of the Web, the distribution of links is no longer a power law but resembles more a log-normal distribution, as observed by Pennock et al. (2002) in the communities of the home pages of universities, public companies, newspapers and scientists. Based on these observations, they propose a generative model that mixes preferential attachment with a baseline probability of gaining a link.

The growth of the networks (adding new nodes) is not a necessary condition for creating a scale-free topology. For instance, it has been shown [2] that Dual Phase Evolution can produce scale-free topologies in networks of a fixed size. Dangalchev (2004) gives examples of generating static scale-free networks. Another possibility (Caldarelli et al. 2002) is to consider the structure as static and draw a link between vertices according to a particular property of the two vertices involved. Once specified the statistical distribution for these vertices properties (fitnesses), it turns out that in some circumstances also static networks develop scale-free properties.

Recently, Manev and Manev (Med. Hypotheses, 2005) proposed that small world networks may be operative in adult brain neurogenesis. Adult neurogenesis has been observed in mammalian brains, including those of humans, but a question remains: how do new neurons become functional in the adult brain? It is proposed that the random addition of only a few new neurons functions as a maintenance system for the brain's "small-world" networks. Randomly added to an orderly network, new links enhance signal propagation speed and synchronizability. Newly generated neurons are ideally suited to become such links: they are immature, form more new connections compared to mature ones, and their number but not their precise location may be maintained by continuous proliferation and dying off. Similarly, it is envisaged that the treatment of brain pathologies by cell transplantation would also create new random links in small-world networks and that even a small number of successfully incorporated new neurons may be functionally important.

## See also

- Social-circles network model - a more generalized generative model for many "real-world networks" of which the scale-free network is a special case
- Random graph
- Bose-Einstein condensation: a network theory approach

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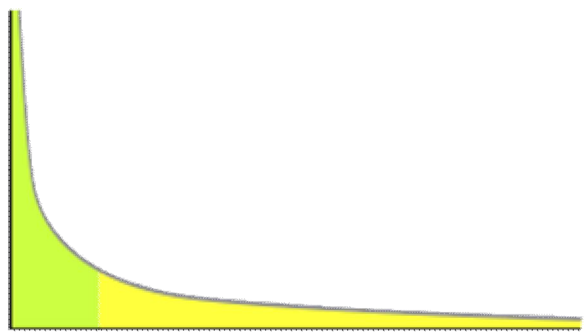
# 3. Power law

An example power law graph, being used to demonstrate ranking of popularity. To the right is the long tail, to the left are the few that dominate (also known as the 80-20 rule).



A **power law** is a special kind of mathematical relationship between two quantities. When the number or frequency of an object or event varies as a power of some attribute of that object (e.g., its size), the number or frequency is said to follow a power law.

For instance, the number of cities having a certain population size is found to vary as a power of the size of the population, and hence follows a power law.

Power laws govern a wide variety of natural and man-made phenomena, including frequencies of words in most languages, frequencies of family names, sizes of craters on the moon and of solar flares, the sizes of power outages, earthquakes, and wars, the popularity of books and music, and many other quantities.

## Technical definition

A power law is any polynomial relationship that exhibits the property of **scale invariance**. The most common power laws relate two variables and have the form

$$f(x) = ax^k + o(x^k),$$

where *a* and *k* are constants, and $o(x^k)$ is an asymptotically small function of $x^k$. Here, *k* is typically called the *scaling exponent*, where the word "scaling" denotes the fact that a power-law function satisfies $f(cx) \propto f(x)$ where

$c$ is a constant. Thus, a rescaling of the function's argument changes the constant of proportionality but preserves the shape of the function itself. This point becomes clearer if we take the logarithm of both sides:

$$\log\left(f(x)\right) = k\log x + \log a.$$

Notice that this expression has the form of a linear relationship with slope $k$. Rescaling the argument produces a linear shift of the function up or down but leaves both the basic form and the slope $k$ unchanged.

Power-law relations characterize a staggering number of naturally occurring phenomena, and this is one of the principal reasons why they have attracted such wide interest. For instance, inverse-square laws, such as gravitation and the Coulomb force, are power laws, as are many common mathematical formulae such as the quadratic law of area of the circle. However much of the recent interest in power laws comes from the study of probability distributions: it's now known that the distributions of a wide variety of quantities seem to follow the power-law form, at least in their upper tail (large events). The behavior of these large events connects these quantities to the study of theory of large deviations (also called extreme value theory), which considers the frequency of extremely rare events like stock market crashes and large natural disasters. It is primarily in the study of statistical distributions that the name "power law" is used; in other areas the power-law functional form is more often referred to simply as a polynomial form or polynomial function.

Scientific interest in power law relations stems partly from the ease with which certain general classes of mechanisms generate them. The demonstration of a power-law relation in some data can point to specific kinds of mechanisms that might underlie the natural phenomenon in question, and can indicate a deep connection with other, seemingly unrelated systems (see the reference by Simon and the subsection on universality below). The ubiquity of power-law relations in physics is partly due to dimensional constraints, while in complex systems, power laws are often thought to be signatures of hierarchy or of specific stochastic processes. A few notable examples of power laws are the Gutenberg-Richter law for earthquake sizes, Pareto's law of income distribution, structural self-similarity of fractals, and scaling laws in biological systems. Research on the origins of power-law relations, and efforts to observe and validate them in the real world, is an active topic of research in many fields of science, including physics, computer science, linguistics, geophysics, sociology, economics and more.

## Properties of power laws

## Scale invariance

The main property of power laws that makes them interesting is their scale invariance. Given a relation $f(x) = ax^k$, scaling the argument $x$ by a constant factor causes only a proportionate scaling of the function itself. That is,

$$f(cx) = a(cx)^k = c^k f(x) \propto f(x).$$

That is, scaling by a constant simply multiplies the original power-law relation by the constant $c^k$. Thus, it follows that all power laws with a particular scaling exponent are equivalent up to constant factors, since each is simply a scaled version of the others. This behavior is what produces the linear relationship when both logarithms are taken of both $f(x)$ and $x$, and the straight-line on the log-log plot is often called the *signature* of a power law. Notably, however, with real data, such straightness is necessary, but not a sufficient condition for the data following a power-law relation. In fact, there are many ways to generate finite amounts of data that mimic this signature behavior, but, in their asymptotic limit, are not true power laws. Thus, accurately fitting and validating power-law models is an active area of research in statistics.

## Universality

The equivalence of power laws with a particular scaling exponent can have a deeper origin in the dynamical processes that generate the power-law relation. In physics, for example, phase transitions in thermodynamic systems are associated with the emergence of power-law distributions of certain quantities, whose exponents are referred to as the critical exponents of the system. Diverse systems with the same critical exponents — that is, which display identical scaling behaviour as they approach criticality — can be shown, via renormalization group theory, to share the same fundamental dynamics. For instance, the behavior of water and $CO_2$ at their boiling points fall in the same universality class because they have identical critical exponents. In fact, almost all material phase transitions are described by a small set of universality classes. Similar observations have been made, though not as comprehensively, for various self-organized critical systems, where the critical point of the system is an attractor. Formally, this sharing of dynamics is referred to as universality, and systems with precisely the same critical exponents are said to belong to the same universality class.

## Power-law functions

The general power-law function follows the polynomial form given above, and is a ubiquitous form throughout mathematics and science. Notably, however, not all polynomial functions are power laws because not all polynomials exhibit the property of scale invariance. Typically, power-law functions are polynomials in a single variable, and are explicitly used to model the scaling behavior of natural processes. For instance, allometric scaling laws for the relation of biological variables are some of the best known power-law functions in nature. In this context, the $o(x^k)$ term is most typically replaced by a deviation term ε, which can represent uncertainty in the observed values (perhaps measurement or sampling errors) or provide a simple way for observations to deviate from the no power-law function (perhaps for stochastic reasons):

$$y = ax^k + \varepsilon.$$

## Examples of power law functions

- The Stevens' power law of psychophysics
- The Stefan–Boltzmann law
- The Ramberg-Osgood stress-strain relationship
- The Inverse-square laws of Newtonian gravity and Electrostatics
- Electrostatic potential and Gravitational potential
- Model of van der Waals force
- Force and potential in Simple harmonic motion
- Kepler's third law
- The Initial mass function
- Gamma correction relating light intensity with voltage
- Kleiber's law relating animal metabolism to size, and allometric laws in general
- Behaviour near second-order phase transitions involving critical exponents
- Proposed form of experience curve effects
- The differential energy spectrum of cosmic-ray nuclei
- Square-cube law (ratio of surface area to volume)
- Constructal law
- Fractals
- The Pareto principle also called the "80-20 rule"
- Zipf's Law in corpus analysis and population distributions amongst others, where frequency of an item or event is inversely proportional to its frequency rank (i.e. the second most frequent item/event occurring half as often the most frequent item and so on).
- Weight vs. length models in fish

## Power-law distributions

A power-law distribution is any that, in the most general sense, has the form

$$p(x) \propto L(x)x^{-\alpha}$$

where $\alpha > 1$, and $L(x)$ is a **slowly varying function**, which is any function that satisfies $\lim_{x\to\infty} L(t\,x)/L(x) = 1$ with $t$ constant. This property of $L(x)$ follows directly from the requirement that $p(x)$ be asymptotically scale invariant; thus, the form of $L(x)$ only controls the shape and finite extent of the lower tail. For instance, if $L(x)$ is the constant function, then we have a power-law that holds for all values of $x$. In many cases, it is convenient to assume a lower bound $x_{min}$ from which the law holds. Combining these two cases, and where $x$ is a continuous variable, the power law has the form

$$p(x) = \frac{\alpha-1}{x_{\min}}\left(\frac{x}{x_{\min}}\right)^{-\alpha},$$

where the pre-factor to $x^{-\alpha}$ is the normalizing constant. We can now consider several properties of this distribution. For instance, its moments are given by

$$\langle x^m \rangle = \int_{x_{\min}}^{\infty} x^m p(x)\,\mathrm{d}x = \frac{\alpha-1}{\alpha-1-m}x_{\min}^m$$

which is only well defined for $m < \alpha - 1$. That is, all moments $m \geq \alpha - 1$ diverge: when $\alpha < 2$, the average and all higher-order moments are infinite; when $2 < \alpha < 3$, the mean exists, but the variance and higher-order moments are infinite, etc. For finite-size samples drawn from such distribution, this behavior implies that the central moment estimators (like the mean and the variance) for diverging moments will never converge - as more data is accumulated, they continue to grow.

Another kind of power-law distribution, which does not satisfy the general form above, is the power law with an exponential cutoff

$$p(x) \propto L(x)x^{-\alpha}\mathrm{e}^{-\lambda x}.$$

In this distribution, the exponential decay term $e^{-\lambda x}$ eventually overwhelms the power-law behavior at very large values of $x$. This distribution does not scale and is thus not asymptotically a power law; however, it does approximately scale over a finite region before the cutoff. (Note that the pure form above is a subset of this family, with $\lambda = 0$.) This distribution is a common alternative to the asymptotic power-law distribution because it naturally captures finite-size effects. For instance, although the Gutenberg–Richter law is commonly cited as an example of a power-law distribution, the distribution of earthquake magnitudes cannot scale as a power law in the limit $x \longrightarrow \infty$ because there is a finite amount of energy in the Earth's crust and thus there must be some maximum size to an earthquake. As the scaling behavior approaches this size, it must taper off.

## Plotting power-law distributions

In general, power-law distributions are plotted on double logarithmic axes, which emphasizes the upper tail region. The most convenient way to do this is via the (complementary) cumulative distribution, $P(x) = \Pr(X > x)$,

$$P(x) = \Pr(X > x) = C \int_x^\infty p(X)\,\mathrm{d}X = \frac{\alpha - 1}{x_{\min}^{-\alpha+1}} \int_x^\infty X^{-\alpha}\,\mathrm{d}X = \left(\frac{x}{x_{\min}}\right)^{-\alpha+1}.$$

Note that the cumulative distribution (cdf) is also a power-law function, but with a smaller scaling exponent. For data, an equivalent form of the cdf is the rank-frequency approach, in which we first sort the $n$ observed values in ascending order, and plot them against the vector $\left[1, \frac{n-1}{n}, \frac{n-2}{n}, \ldots, \frac{1}{n}\right]$.

Although it can be convenient to log-bin the data, or otherwise smooth the probability density (mass) function directly, these methods introduce an implicit bias in the representation of the data, and thus should be avoided. The cdf, on the other hand, introduces no bias in the data and preserves the linear signature on doubly logarithmic axes.

## Estimating the exponent from empirical data

There are many ways of estimating the value of the scaling exponent for a power-law tail, however not all of them yield unbiased and consistent answers. The most reliable techniques are often based on the method of maximum likelihood. Alternative methods are often based on making a linear regression on either the log-log probability, the log-log cumulative distribution function, or on log-binned data, but these approaches should be avoided as they can all lead to highly biased estimates of the scaling exponent (see the Clauset et al. reference below).

For real-valued data, we fit a power-law distribution of the form

$$p(x) = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}}\right)^{-\alpha}$$

to the data $x \geq x_{\min}$. Given a choice for $x_{\min}$, a simple derivation by this method yields the estimator equation

$$\hat{\alpha} = 1 + n \left[\sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}\right]^{-1}$$

where $\{x_i\}$ are the $n$ data points $x_i \geq x_{\min}$. (For a more detailed derivation, see Hall or Newman below.) This estimator exhibits a small finite sample-size bias of order $O(n^{-1})$, which is small when $n > 100$. Further, the uncertainty in the estimation can be derived from the maximum likelihood argument, and has the form $\sigma = \frac{\alpha - 1}{\sqrt{n}}$. This estimator is equivalent to the popular Hill estimator from quantitative finance and extreme value theory.

For a set of $n$ integer-valued data points $\{x_i\}$, again where each $x_i \geq x_{\min}$, the maximum likelihood exponent is the solution to the transcendental equation

$$\frac{\zeta'(\hat{\alpha}, x_{\min})}{\zeta(\hat{\alpha}, x_{\min})} = -\frac{1}{n} \sum_{i=1}^{n} \ln \frac{x_i}{x_{\min}}$$

where $\zeta(\alpha, x_{\min})$ is the incomplete zeta function. The uncertainty in this estimate follows the same formula as for the continuous equation. However, the two equations for $\hat{\alpha}$ are not equivalent, and the continuous version should not be applied to discrete data, nor vice versa.

Further, both of these estimators require the choice of $x_{\min}$. For functions with a non-trivial $L(x)$ function, choosing $x_{\min}$ too small produces a significant bias in $\hat{\alpha}$, while choosing it too large increases the uncertainty in $\hat{\alpha}$, and reduces the statistical power of our model. In general, the best choice of $x_{min}$ depends strongly on the particular form of the lower tail, represented by $L(x)$ above.

More about these methods, and the conditions under which they can be used, can be found in the Clauset et al. reference below. Further, this comprehensive review article provides usable code (Matlab and R) for estimation and testing routines for power-law distributions.

## Examples of power-law distributions

- Pareto distribution (continuous)
- Zeta distribution (discrete)
- Yule–Simon distribution (discrete)
- Student's t-distribution (continuous), of which the Cauchy distribution is a special case
- Zipf's law and its generalization, the Zipf-Mandelbrot law (discrete)
    - Lotka's law
- The scale-free network model
- Bibliograms
- Gutenberg–Richter law of earthquake magnitudes
- Horton's laws describing river systems
- Richardson's Law for the severity of violent conflicts (wars and terrorism)
- population of cities
- numbers of religious adherents
- net worth of individuals
- frequency of words in a text
- Pink noise
- 90-9-1 principle on wikis

A great many power-law distributions have been conjectured in recent years. For instance, power laws are thought to characterize the behavior of the upper tails for the popularity of websites, number of species per genus, the popularity of given names, the size of financial returns, and many others. However, much debate remains as to which of these tails are actually power-law distributed and which are not. For instance, it is commonly accepted now that the famous Gutenberg–Richter law decays more rapidly than a pure power-law tail because of a finite exponential cutoff in the upper tail.

## Validating power laws

Although power-law relations are attractive for many theoretical reasons, demonstrating that data do indeed follow a power-law relation requires more than simply fitting such a model to the data. In general, many alternative functional forms can appear to follow a power-law form for some extent. Thus, the preferred method for validation of power-law relations is by testing many orthogonal predictions of a particular generative mechanism against data, and not simply fitting a power-law relation to a particular kind of data. As such, the validation of power-law claims remains a very active field of research in many areas of modern science.[citation needed]

## See also

- Fat tail
- Heavy-tailed distributions
- Lévy flight
- Lognormal distribution
- The Long Tail
- Power law fluid
- Simon Model
- stable distribution
- Stevens' power law
- Wealth condensation
- The following are mentioned elsewhere in the article:
    - 80-20 rule (graph at the top, Examples of power-law distributions)
    - Allometric law (Power-law functions, Examples of power-law distributions)
    - Extreme value theory (Technical definition, Estimating the exponent from empirical data)
    - Kleiber's law (text at the top, Examples of power-law distributions)
    - Zipf's law (Examples of power-law distributions)

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- *Ubiquity* Mark Buchanan (2000) Wiedenfield & Nicholson ISBN 0 297 64376 2


## External links

- Zipf's law
- Power laws, Pareto distributions and Zipf's law
- Zipf, Power-laws, and Pareto - a ranking tutorial
- Gutenberg-Richter Law
- Stream Morphometry and Horton's Laws
- Clay Shirky on Institutions & Collaboration: Power law in relation to the internet-based social networks
- Clay Shirky on Power Laws, Weblogs, and Inequality
- "How the Finance Gurus Get Risk All Wrong" by Benoit Mandelbrot & Nassim Nicholas Taleb. *Fortune*, July 11, 2005.
- "Million-dollar Murray": power-law distributions in homelessness and other social problems; by Malcolm Gladwell. *The New Yorker*, February 13, 2006.
- Benoit Mandelbrot & Richard Hudson: The Misbehaviour of Markets (2004)
- Philip Ball: Critical Mass: How one thing leads to another (2005)
- *Tyranny of the Power Law* from The Econophysics Blog
- *So You Think You Have a Power Law — Well Isn't That Special?* from Three-Toed Sloth, the blog of Cosma Shalizi, Professor of Statistics at Carnegie-Mellon University.

# 4. Pareto principle

The **Pareto principle** (also known as the **80-20 rule**,[1] the **law of the vital few,** and the **principle of factor sparsity**) states that, for many events, roughly 80% of the effects come from 20% of the causes.[2][3] Business management thinker Joseph M. Juran suggested the principle and named it after Italian economist Vilfredo Pareto, who observed in 1906 that 80% of the land in Italy was owned by 20% of the population; he developed the principle by observing that 20% of the pea pods in his garden contained 80% of the peas.[3] It is a common rule of thumb in business; e.g., "80% of your sales come from 20% of your clients." Mathematically, where something is shared among a sufficiently large set of participants, there must be a number $k$ between 50 and 100 such that $k$% is taken by (100 – $k$)% of the participants. $k$ may vary from 50 (in the case of equal distribution) to nearly 100 (when a tiny number of participants account for almost all of the resource). There is nothing special about the number 80% mathematically, but many real systems have $k$ somewhere around this region of intermediate imbalance in distribution.

The Pareto principle is only tangentially related to Pareto efficiency, which was also introduced by the same economist. Pareto developed both concepts in the context of the distribution of income and wealth among the population.

## In economics

The original observation was in connection with income and wealth. Pareto noticed that 80% of Italy's wealth was owned by 20% of the population.[4] He then carried out surveys on a variety of other countries and found to his surprise that a similar distribution applied.

Because of the scale-invariant nature of the power law relationship, the relationship applies also to subsets of the income range. Even if we take the ten wealthiest individuals in the world, we see that the top three (Warren Buffett, Carlos Slim Helú, and Bill Gates) own as much as the next seven put together.[5]

A chart that gave the inequality a very visible and comprehensible form, the so-called 'champagne glass' effect,[6] was contained in the 1992 United Nations Development Program Report, which showed the distribution of global income to be very uneven, with the richest 20% of the world's population controlling 82.7% of the world's income.[7]

| Distribution of world GDP, 1989[8] | |
|---|---|
| **Quintile of population** | **Income** |
| Richest 20% | 82.70% |
| Second 20% | 11.75% |
| Third 20% | 2.30% |
| Fourth 20% | 1.85% |
| Poorest 20% | 1.40% |

The Pareto Principle has also been used to attribute the widening economic inequality in the USA to 'skill-biased technical change' – i.e. income growth accrues to those with the education and skills required to take advantage of new technology and globalisation. However, Nobel Prize winner in Economics Paul Krugman in the *New York Times* dismissed this "80-20 fallacy" as being cited "not because it's true, but because it's comforting." He asserts that the benefits of economic growth over the last 30 years have largely been concentrated in the top 1%, rather than the top 20%[9].

## In software

In computer science and engineering control theory such as for electromechanical energy converters, the Pareto principle can be applied to optimization efforts.[10]

Microsoft also noted that by fixing the top 20% of the most reported bugs, 80% of the errors and crashes would be eliminated.[11]

In computer graphics the Pareto principle is used for ray-tracing: 80% of rays intersect 20% of geometry.[12]

## Other applications

In the systems science discipline, Epstein and Axtell created an agent-based simulation model called Sugarscape, from a decentralized modeling approach, based on individual behavior rules defined for each agent in the economy. Wealth distribution and Pareto's 80/20 Principle became emergent in their results, which suggests that the principle is a natural phenomenon.[13]

The Pareto Principle also applies to a variety of more mundane matters: one might guess approximately that we wear our 20% most favoured clothes about 80% of the time, perhaps we spend 80% of the time with 20% of our acquaintances, etc.

The Pareto principle has many applications in quality control.[*citation needed*] It is the basis for the Pareto chart, one of the key tools used in total quality control and six sigma. The Pareto principle serves as a baseline for ABC-analysis and XYZ-analysis, widely used in logistics and procurement for the purpose of optimizing stock of goods, as well as costs of keeping and replenishing that stock.[14]

The Pareto principle was a prominent part of the 2007 bestseller *The 4-Hour Workweek* by Tim Ferriss. Ferriss recommended focusing one's attention on those 20% that contribute to 80% of the income. More notably, he also recommends firing those 20% of customers who take up the majority of one's time and cause the most trouble.[15]

In human developmental biology the principle is reflected in the gestation period where the embryonic period constitutes 20% of the whole, with the foetal development taking up the rest of the time.

In health care in the United States, it has been found that 20% of patients use 80% of health care resources.[16]

## Mathematical notes

The idea has rule-of-thumb application in many places, but it is commonly misused. For example, it is a misuse to state that a solution to a problem "fits the 80-20 rule" just because it fits 80% of the cases; it must be implied that this solution requires only 20% of the resources needed to solve all cases. Additionally, it is a misuse of the 80-20 rule to interpret data with a small number of categories or observations.

Mathematically, where something is shared among a sufficiently large set of participants, there will always be a number $k$ between 50 and 100 such that $k$% is taken by (100 – $k$)% of the participants; however, $k$ may vary from 50 in the case of equal distribution (e.g. exactly 50% of the people take 50% of the resources) to nearly 100 in the case of a tiny number of participants taking almost all of the resources. There is nothing special about the number 80, but many systems will have $k$ somewhere around this region of intermediate imbalance in distribution.

This is a special case of the wider phenomenon of Pareto distributions. If the parameters in the Pareto distribution are suitably chosen, then one would have not only 80% of effects coming from 20% of causes, but also 80% of that top 80% of effects coming from 20% of that top 20% of causes, and so on (80% of 80% is 64%; 20% of 20% is 4%, so this implies a "64-4" law; and a similarly implies a "51.2-0.8" law).

80-20 is only a shorthand for the general principle at work. In individual cases, the distribution could just as well be, say, 80-10 or 80-30. (There is no need for the two numbers to add up to 100%, as they are measures of different things, e.g., 'number of customers' vs 'amount spent'). The classic 80-20 distribution occurs when the gradient of the line is –1 when plotted on log-log axes of equal scaling. Pareto rules are not mutually exclusive. Indeed, the 0-0 and 100-100 rules always hold.

Adding up to 100 leads to a nice symmetry. For example, if 80% of effects come from the top 20% of sources, then the remaining 20% of effects come from the lower 80% of sources. This is called the "joint ratio", and can be used to measure the degree of imbalance: a joint ratio of 96:4 is very imbalanced, 80:20 is significantly imbalanced (Gini index: 60%), 70:30 is moderately imbalanced (Gini index: 40%), and 55:45 is just slightly imbalanced.

The Pareto Principle is an illustration of a "power law" relationship, which also occurs in phenomena such as brush-fires and earthquakes.[citation needed] Because it is self-similar over a wide range of magnitudes, it produces outcomes completely different from Gaussian distribution phenomena. This fact explains the frequent breakdowns of sophisticated financial instruments, which are modeled on the assumption that a Gaussian relationship is appropriate to—for example—stock movement sizes.[17]

## Equality measures

### Gini coefficient and Hoover index

Using the "*A* : *B*" notation (for example, 0.8:0.2) and with *A* + *B* = 1, inequality measures like the Gini index *and* the Hoover index can be computed. In this case both are the same.

$$H = G = |2A - 1| = |2B - 1|$$

$$A : B = \left(\frac{1+H}{2}\right) : \left(\frac{1-H}{2}\right)$$

### Theil index

The Theil index is an entropy measure used to quantify inequities. The measure is 0 for 50:50 distributions and reaches 1 at a Pareto distribution of 82:18. Higher inequities yield Theil indices above 1.[18]

$$T_T = T_L = T_s = 2H\,\mathrm{arctanh}\,(H)\,.$$

## See also

- 10/90 gap
- Benford's law
- Mathematical economics
- Sturgeon's law
- The Long Tail
- Vitality curve
- Wealth condensation
- Zipf's law
- Principle of least effort
- Richard Koch
- Ninety-ninety rule
- Parkinson's law
- 1% rule
- Elephant flow

## Examples

- Megadiverse countries

## Notes

1. ^ The Pareto principle has several name variations, including: Pareto's Law, the 80/20 rule, the 80:20 rule, and 80 20 rule.
2. ^ Bunkley, Nick (March 3, 2008), "Joseph Juran, 103, Pioneer in Quality Control, Dies", *New York Times*, http://www.nytimes.com/2008/03/03/business/03juran.html.
3. ^ a b *What is 80/20 Rule, Pareto's Law, Pareto Principle* http://www.80-20presentationrule.com/whatisrule.html.
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15. ^ Ferris, Tim (2007), *The 4-Hour Workweek*, Crown Publishing.
16. ^ http://www.projo.com/opinion/contributors/content/CT_weinberg27_07-27-09_HQF0P1E_v15.3f89889.html
17. ^ Taleb, Nassim (2007). *The Black Swan*. pp. 228–252, 274–285.
18. ^ On Line Calculator: Inequality

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## External links

- About.com: Pareto's Principle
- Wealth Condensation in Pareto Macro-Economies

# 5. Natural monopoly

In economics, a **natural monopoly** occurs when, due to the economies of scale of a particular industry, the maximum efficiency of production and distribution is realized through a single supplier, but in some cases inefficiency may take place.

Natural monopolies arise where the largest supplier in an industry, often the first supplier in a market, has an overwhelming cost advantage over other actual or potential competitors. This tends to be the case in industries where capital costs predominate, creating economies of scale which are large in relation to the size of the market, and hence high barriers to entry; examples include public utilities such as water services and electricity. It is very expensive to build transmission networks (water/gas pipelines, electricity and telephone lines), therefore it is unlikely that a potential competitor would be willing to make the capital investment needed to even enter the monopolist's market.

It may also depend on control of a particular natural resource. Companies that grow to take advantage of economies of scale often run into problems of bureaucracy; these factors interact to produce an "ideal" size for a company, at which the company's average cost of production is minimized. If that ideal size is large enough to supply the whole market, then that market is a natural monopoly.

Some free-market-oriented economists argue that natural monopolies exist only in theory, and not in practice, or that they exist only as transient states.[1]

## Explanation

All industries have costs associated with entering them. Often, a large portion of these costs is required for investment. Larger industries, like utilities, require enormous initial investment. This barrier to entry reduces the number of possible entrants into the industry regardless of the earning of the corporations within. Natural monopolies arise where the largest supplier in an industry, often the first supplier in a market, has an overwhelming cost advantage over other actual or potential competitors; this tends to be the case in industries where fixed costs predominate, creating economies of scale which are large in relation to the size of the market - examples include water services and electricity. It is very expensive to build transmission networks (water/gas pipelines, electricity and telephone lines), therefore it is unlikely that a potential competitor would be willing to make the capital investment needed to even enter the monopolist's market.

Companies that grow to take advantage of economies of scale often run into problems of bureaucracy; these factors interact to produce an "ideal" size for a company, at which the company's average cost of production is minimized. If that ideal size is large enough to supply the whole market, then that market is a natural monopoly.

A further discussion and understanding requires more microeconomics:

Two different types of cost are important in microeconomics: **marginal cost**, and **fixed cost**. The marginal cost is the cost to the company of serving one more customer. In an industry where a natural monopoly does not exist, the vast majority of industries, the marginal cost decreases with economies of scale, then increases as the company has growing pains (overworking its employees, bureaucracy, inefficiencies, etc.). Along with this, the average cost of its products will decrease and then increase again. A natural monopoly has a very different cost structure. A natural monopoly has a high fixed cost for a product that does not depend on output, but its marginal cost of producing one more good is roughly constant, and small.

A firm with high fixed costs will require a large number of customers in order to retrieve a meaningful return on their initial investment. This is where economies of scale become important. Since each firm has large initial costs, as the firm gains market share and increases its output the fixed cost (what they initially invested) is divided among a larger number of customers. Therefore, in industries with large initial investment requirements, average total cost declines as output increases over a much larger range of output levels.

Once a natural monopoly has been established because of the large initial cost and that, according to the rule of economies of scale, the larger corporation (to a point) has lower average cost and therefore a huge advantage. With this knowledge, no firms attempt to enter the industry and an oligopoly or monopoly develops.

## Industries with a natural monopoly

Utilities are often natural monopolies. In industries with a standardized product and economies of scale, a natural monopoly will often arise. In the case of electricity, all companies provide the same product, the infrastructure required is immense, and the cost of adding one more customer is negligible, up to a point. Adding one more customer may increase the company's revenue and lowers the average cost of providing for the company's customer base. So long as the average cost of serving customers is decreasing, the larger firm will more efficiently serve the entire customer base. Of course, this might be circumvented by differentiating the product, making it no longer a pure commodity. For example, firms may gain customers who will pay more by selling "green" power, or non-polluting power, or locally-produced power.

## Historical example

Such a process happened in the water industry in nineteenth century Britain. Up until the mid-nineteenth century, Parliament discouraged municipal involvement in water supply; in 1851, private companies had 60% of the market.

Competition amongst the companies in larger industrial towns lowered profit margins, as companies were less able to charge a sufficient price for installation of networks in new areas. In areas with direct competition (with two sets of mains), usually at the edge of companies' territories, profit margins were lowest of all.

Such situations resulted in higher costs and lower efficiency, as two networks, neither used to capacity, were used. With a limited number of households that could afford their services, expansion of networks slowed, and many companies were barely profitable. With a lack of water and sanitation claiming thousands of lives in periodic epidemics, municipalisation proceeded rapidly after 1860, and it was municipalities which were able to raise the finance for investment which private companies in many cases could not.

A few well-run private companies which worked together with their local towns and cities (gaining legal monopolies and thereby the financial security to invest as required) did survive, providing around 20% of the population with water even today.

The rest of the water industry in England and Wales was reprivatised in the form of 10 regional monopolies in 1989.

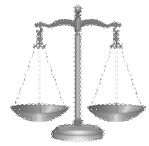
## Origins of the term

The original concept of natural monopoly is often attributed to John Stuart Mill, who (writing before the marginalist revolution) believed that prices would reflect the costs of production in absence of an artificial or natural monopoly.[2] In *Principles of Political Economy* Mill criticised Smith's neglect[3] of an area that could explain wage disparity. Taking up the examples of professionals such as jewellers, physicians and lawyers, he said,[4]

"The superiority of reward is not here the consequence of competition, but of its absence: not a compensation for disadvantages inherent in the employment, but an extra advantage; a kind of monopoly price, the effect not of a legal, but of what has been termed a natural monopoly... independently of... artificial monopolies [i.e. grants by government], there is a natural monopoly in favour of skilled labourers against the unskilled, which makes the difference of reward exceed, sometimes in a manifold proportion, what is sufficient merely to equalize their advantages. If unskilled labourers had it in their power to compete with skilled, by merely taking the trouble of learning the trade, the difference of wages might not exceed what would compensate them for that trouble, at the ordinary rate at which labour is remunerated. But the fact that a course of instruction is required, of even a low degree of costliness, or that the labourer must be maintained for a considerable time from other sources, suffices everywhere to exclude the great body of the labouring people from the possibility of any such competition.

So Mill's initial use of the term concerned natural abilities, in contrast to the common contemporary usage, which refers solely to market failure in a particular type of industry, such as rail, post or electricity. Mill's development of the idea is that what is true of labour is true of capital.[5]

"All the natural monopolies (meaning thereby those which are created by circumstances, and not by law) which produce or aggravate the disparities in the remuneration of different kinds of labour, operate similarly between different employments of capital. If a business can only be advantageously carried on by a large capital, this in most countries limits so narrowly the class of persons who can enter into the employment, that they are enabled to keep their rate of profit above the general level. A trade may also, from the nature of the case, be confined to so few hands, that profits may admit of being kept up by a combination among the dealers. It is well known that even among so numerous a body as the London booksellers, this sort of combination long continued to exist. I have already mentioned the case of the gas and water companies.



**Competition law**

**Basic concepts**

- History of competition law
- Monopoly
  - Coercive monopoly
  - Natural monopoly
- Barriers to entry
- Market power
- SSNIP test
- Relevant market
- Merger control

**Anti-competitive practices**

- Monopolization
- Collusion
  - Formation of cartels
  - Price fixing
  - Bid rigging
- Product bundling and tying
- Refusal to deal
  - Group boycott
  - Essential facilities
- Exclusive dealing
- Dividing territories
- Conscious parallelism
- Predatory pricing
- Misuse of patents and copyrights

**Enforcement authorities and organizations**

- International Competition Network
- List of competition regulators

This box: view • talk • edit

Mill also used the term in relation to land, for which the natural monopoly could be extracted by virtue of it being the only land like it.[6] Furthermore, Mill referred to network industries, such as electricity and water supply, roads, rail and canals, as "practical monopolies", where "it is the part of the government, either to subject the business to reasonable conditions for the general advantage, or to retain such power over it, that the profits of the monopoly may at least be obtained for the public."[7][8] So, a legal prohibition against competition is often advocated and rates are not left to the market but are regulated by the government.

## Regulation

As with all monopolies, a monopolist who has gained his position through natural monopoly effects may engage in behavior that abuses his market position, which often leads to calls from consumers for government regulation.

Government regulation may also come about at the request of a business hoping to enter a market otherwise dominated by a natural monopoly.

Common arguments in favor of regulation include the desire to control market power, facilitate competition, promote investment or system expansion, or stabilize markets. In general, though, regulation occurs when the government believes that the operator, left to his own devices, would behave in a way that is contrary to the government's objectives. In some countries an early solution to this perceived problem was government provision of, for example, a utility service. However, this approach raised its own problems. Some governments used the state-provided utility services to pursue political agendas, as a source of cash flow for funding other government activities, or as a means of obtaining hard currency. These and other consequences of state provision of services often resulted in inefficiency and poor service quality. As a result, governments began to seek other solutions, namely regulation and providing services on a commercial basis, often through private participation.[9]

As a quid pro quo for accepting government oversight, private suppliers may be permitted some monopolistic returns, through stable prices or guaranteed through limited rates of return, and a reduced risk of long-term competition. (See also rate of return pricing). For example, an electric utility may be allowed to sell electricity at price that will give it a 12% return on its capital investment. If not constrained by the public utility commission, the company would likely charge a far higher price and earn an abnormal profit on its capital.

Regulatory responses:

- doing nothing
- setting legal limits on the firm's behaviour, either directly or through a regulatory agency
- setting up competition *for* the market (franchising)
- setting up common carrier type competition
- setting up surrogate competition ("yardstick" competition or benchmarking)
- requiring companies to be (or remain) quoted on the stock market
- public ownership

Since the 1980s there is a global trend towards utility deregulation, in which systems of competition are intended to replace regulation by specifying or limiting firms' behaviour; the telecommunications industry is a leading example globally.

## Doing nothing

Because the existence of a natural monopoly depends on an industry's cost structure, which can change dramatically through new technology (both physical and organizational/institutional), the nature or even existence of natural monopoly may change over time. A classic example is the undermining of the natural monopoly of the canals in eighteenth century Britain by the emergence in the nineteenth century of the new technology of railways.

Arguments from public choice suggest that regulatory capture is likely in the case of a regulated private monopoly. Moreover, in some cases the costs to society of overzealous regulation may be higher than the costs of permitting an unregulated private monopoly. (Although the monopolist charges monopoly prices, much of the price increase is a transfer rather than a loss to society.)

More fundamentally, the theory of contestable markets developed by Baumol and others argues that monopolists (including natural monopolists) may be forced over time by the mere *possibility* of competition at some point in the future to limit their monopolistic behaviour, in order to deter entry. In the limit, a monopolist is forced to make the same production decisions as a competitive market would produce. A common example is that of airline flight schedules, where a particular airline may have a monopoly between destinations A and B, but the relative ease with which in many cases competitors could also serve that route limits its monopolistic behaviour. The argument even applies somewhat to government-granted monopolies, as although they are protected from competitors entering the industry, in a democracy excessively monopolistic behaviour may lead to the monopoly being revoked, or given to another party.

Nobel economist Milton Friedman, said that in the case of natural monopoly that "there is only a choice among three evils: private unregulated monopoly, private monopoly regulated by the state, and government operation." He said "the least of these evils is private unregulated monopoly where this is tolerable." He reasons that the other alternatives are "exceedingly difficult to reverse," and that the dynamics of the market should be allowed the opportunity to have an effect and are likely to do so (*Capitalism and Freedom*). In a Wincott Lecture, he said that if the commodity in question is "essential" (for example: water or electricity) and the "monopoly power is sizeable," then "either public regulation or ownership may be a lesser evil." However, he goes on to say that such action by government should not consist of forbidding competition by law. Friedman has taken a stronger laissez-faire stance since, saying that "over time I have gradually come to the conclusion that antitrust laws do far more harm than good and that we would be better off if we didn't have them at all, if we could get rid of them" (*The Business Community's Suicidal Impulse*).

Advocates of laissez-faire capitalism, such as libertarians, typically say that permanent natural monopolies are merely theoretical. Economists from the Austrian school claim that governments take ownership of the means of production in certain industries and ban competition under the false pretense that they are natural monopolies.[10]

## Franchising and outsourcing

Although competition within a natural monopoly market is costly, it is possible to set up competition *for* the market. This has been, for example, the dominant organizational method for water services in France, although in this case the resulting degree of competition is limited by contracts often being set for long periods (30 years), and there only being three major competitors in the market.

Equally, competition may be used for *part* of the market (e.g. IT services), through outsourcing contracts; some water companies outsource a considerable proportion of their operations. The extreme case is Welsh Water, which outsources virtually its entire business operations, running just a skeleton staff to manage these contracts. Franchising different parts of the business on a regional basis (e.g. parts of a city) can bring in some features of "yardstick" competition (see below), as the performance of different contractors can be compared. See also water privatization.

## Common carriage competition

This involves different firms competing to distribute goods and services via the same infrastructure - for example different electricity companies competing to provide services to customers over the same electricity network. For this to work requires government intervention to break up vertically integrated monopolies, so that for instance in electricity, generation is separated from distribution and possibly from other parts of the industry such as sales. The key element is that access to the network is available to any firm that needs it to supply its service, with the price the infrastructure owner is permitted to charge being regulated. (There are several competing models of network access pricing.) In the British model of electricity liberalization, there is a market for generation capacity, where electricity can be bought on a minute-to-minute basis or through longer-term contracts, by companies with insufficient generation capacity (or sometimes no capacity at all).

Such a system may be considered a form of deregulation, but in fact it requires active government creation of a new system of competition rather than simply the removal of existing legal restrictions. The system may also need continuing government finetuning, for example to prevent the development of long-term contracts from reducing the liquidity of the generation market too much, or to ensure the correct incentives for long-term security of supply are present. See also California electricity crisis. Whether such a system is more efficient than possible alternatives is unclear; the cost of the market mechanisms themselves are substantial, and the vertical de-integration required introduces additional risks. This raises the cost of finance - which for a capital intensive industry (as natural monopolies are) is a key issue. Moreover, such competition also raises equity and efficiency issues, as large industrial consumers tend to benefit much more than domestic consumers.

## Stock market

One regulatory response is to require that private companies running natural monopolies be quoted on the stock market. This ensures they are subject to certain financial transparency requirements, and maintains the possibility of a takeover if the company is mismanaged. The latter in theory should help ensure that company is efficiently run. By way of example, the UK's water economic regulator, Ofwat, sees the stock market as an important regulatory instrument for ensuring efficient management of the water companies.

In practice, the notorious short-termism of the stock market may be antithetical to appropriate spending on maintenance and investment in industries with long time horizons, where the failure to do so may only have effects a decade or more hence (which is typically long after current chief executives have left the company).

## Public ownership

A traditional solution to the regulation problem, especially in Europe, is public ownership. This 'cuts out the middle man': instead of government regulating a firm's behaviour, it simply takes it over (usually by buy-out), and sets itself limits within which to act.

## Network effects

Network effects are considered separately from natural monopoly status. Natural monopoly effects are a property of the producer's cost curves, whilst network effects arise from the benefit to the consumers of a good from standardization of the good. Many goods have both properties, like operating system software and telephone networks.

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## See also

- Market forms
- Currency
- Standardization
- Public goods
- Anti-competitive practices
- Coercive monopoly
- Tipping point
- Quasi-rent
- LoopCo

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## External links

- Natural Monopoly Definition

# 6. The rich get richer and the poor get poorer

**"The rich get richer and the poor get poorer"** is a catchphrase and proverb, frequently used (with variations in wording) in discussing economic inequality.

## Predecessors

Andrew Jackson, in his 1832 bank veto, said that

when the laws undertake... to make the rich richer and the potent more powerful, the humble members of society... have a right to complain of the injustice to their Government.[1][2]

William Henry Harrison said, in an October 1, 1840 speech, I believe and I say it is true Democratic feeling, that all the measures of the government are directed to the purpose of making the rich richer and the poor poorer.[3][4]

In 1821, Percy Bysshe Shelley argued, in A Defence of Poetry (not published until 1840), that in his England, "the promoters of utility" had managed

to exasperate at once the extremes of luxury and want. They have exemplified the saying, “To him that hath, more shall be given; and from him that hath not, the little that he hath shall be taken away.” The rich have become richer, and the poor have become poorer; and the vessel of the State is driven between the Scylla and Charybdis of anarchy and despotism. Such are the effects which must ever flow from an unmitigated exercise of the calculating faculty.[5]

The phrase resembles the Bible verse

For whosoever hath, to him shall be given, and he shall have more abundance: but whosoever hath not, from him shall be taken away even that he hath.[6]

However, in this verse Jesus is not referring to economic inequality at all. Rather it is part of his answer to the question "Why speakest thou unto them in parables?" Jesus says his parables give fresh understanding only to those who already have accepted his message.

## "Ain't We Got Fun"

A version of the phrase was popularized in 1921 in the wildly successful song Ain't We Got Fun, and the phrase sometimes attributed to the song's lyricists, Gus Kahn and Raymond B. Egan. Oddly, the lyrics never actually say that the poor get "poorer;" instead it takes off from or alludes to the line, showing that it was already proverbial. They cue the listener to expect the word "poorer," but instead say

*There's nothing surer: The rich get rich and the poor get—children;*

and, later:

*There's nothing surer: The rich get rich and the poor get laid off;*

Note too that the Kahn and Egan lyrics say "the rich get *rich,*" not *richer.*[7][8][9]

The line is sometimes mistakenly attributed to F. Scott Fitzgerald. It appears in The Great Gatsby, as

the rich get richer[sic] and the poor get—children!

The character Gatsby orders the character Klipspringer, sitting at the piano, "Don't talk so much, old sport.... "Play!" and Klipsprinter breaks into the Kahn and Egan song.[10]

## In political and economic rhetoric

The line is often cited by opponents of capitalism as a statement of fact and by supporters of capitalism as an example of an erroneous belief. Thus,

The modern-day statistical work of Stanley Lebergott and Michael Cox confirms this Smithian view and disputes the commonly held criticism that under a free market the rich get richer and the poor get poorer.[11]

According to Marx, capitalism will inevitably lead to ruin in accordance with certain laws of economic movement. These laws are "the Law of the Tendency of the Rate of Profit to Fall," "the Law of Increasing Poverty," and "the Law of Centralization of Capital."[12] Small capitalists go bankrupt, and their production means are absorbed by large capitalists. During the process of bankruptcy and absorption, capital is gradually centralized by a few large capitalists, and the entire middle class declines. Thus, two major classes, a small minority of large capitalists, and a large proletarian majority are formed.[13]

A use of the phrase by a free market advocates disputing the claim is:

Relative cohort inequity decreased markedly, with the poor improving their position much faster than the rich. Relative percentile inequity increased slightly. In terms of buying power, both the poor cohort and the poor percentile became significantly wealthier. These data indicate that the view that the rich are getting richer and the poor are getting poorer is clearly over-generalised.[14]

In the United States the phrase has been used frequently (in the past tense) to describe alleged socioeconomic trends under the Ronald Reagan and George H. W. Bush presidencies.[15][16][17] Defenders of the Reagan policies characterize this claim as inciting class warfare.[18]

Commentators refer to the idea as a cliché in discussions of economic inequality, but one that they argue to be accurate nonetheless:

It's a cliché, perhaps, to say that "the rich get rich and the poor get poorer". But in the 1980s and 1990s, cliché or not, that is what took place in some regions of the world, particularly in South Asia and sub-Saharan Africa.[19]

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3. ^ "William Henry Harrison quotes". James Richard Howington (personal web page). http://home.att.net/~howington/whh.html. Retrieved 2006-08-12.
4. ^ Degregorio, William (1997). *Complete Book of U.S. Presidents: From George Washington to George W. Bush*. Gramercy. ISBN 0-517-18353-6., p. 146; quotes "all the measures of the government are directed to the purpose of making the rich richer and the poor poorer" and sources it to Schlesinger, Arthur (1946). *The Age of Jackson*. Boston: Little, Brown., p. 292
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## Further reading


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## 7. Ain't We Got Fun?

*For the 1937 Merrie Melodies cartoon, see Ain't We Got Fun (cartoon).*

**"Ain't We Got Fun?"** is a popular foxtrot published in 1921 with music by Richard A. Whiting, lyrics by Raymond B. Egan and Gus Kahn.

It was first performed in 1920 in the revue *Satires of 1920*, then moved into vaudeville and recordings. "Ain't We Got Fun?" and both its jaunty response to poverty and its promise of fun "Every morning / Every evening", and "In the meantime, / In between time" have become symbolic of the Roaring Twenties, and it appears in some of the major literature of the decade, including *The Great Gatsby* by F. Scott Fitzgerald and in Dorothy Parker's award-winning short story of 1929, "Big Blonde".

### Composition

"Ain't We Got Fun" follows the structure of a foxtrot.[1] The melody uses mainly quarter notes, and has an unsyncopated refrain made up largely of variations on a repeated four-note phrase.[2][3][4] The *Tin Pan Alley Song Encyclopedia* describes it as a "Roaring Twenties favourite" and praises its vibrancy, "zesty music", and comic lyrics.[5]

Philip Furia, connecting Kahn's lyrics to the song's music, writes that:

Not only does Kahn use abrupt, colloquial--even ungrammatical--phrases, he abandons syntax for the telegraphic connections of conversation. Truncated phrases like *not much money* are the verbal equivalent of syncopated musical fragments.[6]

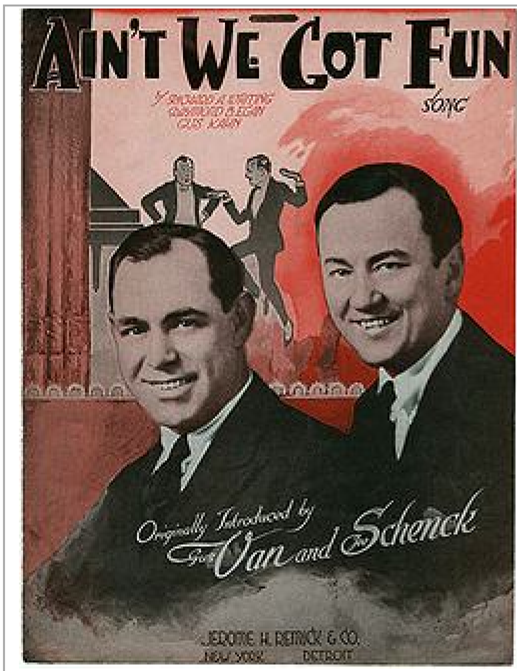
—Philip Furia, *The Poets of Tin Pan Alley: A History of America's Great Lyricists*

Critical appraisals vary regarding what view of poverty the song's lyrics take. Nicholas E. Tawa summarizes the refrain *Ain't we got fun* as a *satirical and jaunty rejoinder* toward hard times.[2] Diane Holloway and Bob Cheney, authors of *American History in Song: Lyrics from 1900 to 1945*, concur, and describe the black humor in the couple's relief that their poverty shields them from worrying about damage to their nonexistent Pierce Arrow luxury automobile.[7]

Yet George Orwell highlights the lyrics of "Ain't We Got Fun" as an example of working class unrest:

All through the war and for a little time afterwards there had been high wages and abundant employment; things were now returning to something worse than normal, and naturally the working class resisted. The men who had fought had been lured into the army by gaudy promises, and they were coming home to a world where there were no jobs and not even any houses. Moreover, they had been at war and were coming home with a soldier's attitude to life, which is fundamentally, in spite of discipline, a lawless attitude. There was a turbulent feeling in the air.[8]

—George Orwell, *The Road to Wigan Pier*




Cover page to the sheet music.

Ain't We Got Fun?

Recorded by Billy Jones in 1921 for Edison Records.

After quoting a few of the song's lines Orwell refers to the era as a time when *people had not yet settled down to a lifetime of unemployment mitigated by endless cups of tea,* a turn of phrase which the later writer Larry Portis contests.[8][9]

He [Orwell] could just as easily have concluded that the song revealed a certain fatalism, a resignation and even capitulation to forces beyond the control of working people. Indeed, it might be only a small step from saying, "Ain't we got fun" in the midst of hardship to the idea that the poor are happier than the rich--because, as the Beatles intoned, "Money can't buy me love." It is possible that "Aint We Got Fun", a product of the music industry (as opposed to 'working-class culture') was part of a complex resolution of crisis in capitalist society. Far from revealing the indomitable spirit of working people, it figured into the means with which they were controlled. It is a problem of interpretation laying at the heart of popular music, one which emerged with particular clarity at the time of the English Industrial Revolution.

—Larry Portis, Soul Trains

However, others concentrate on the fun that they got. Stephen J. Whitfield, citing lyrics such as "Every morning / Every evening / Ain't we got fun", writes that the song "set the mood which is indelibly associated with the Roaring Twenties", a decade when pleasure was sought and found constantly, morning, evening, and "In the meantime / In between time".[10] Philip Furia and Michael Lasser see implicit references to sexual intercourse in lyrics such as *the happy chappy, and his bride of only a year.*[11] Looked at in the context of the 1920s, an era of increasing sexual freedom, they point out that, while here presented within the context of marriage (in other songs it is not), the sexuality is notably closer to the surface than in previous eras and is presented as a delightful, youthful pleasure.[11][12]

There are several variations on the lyrics. For example, *American History in Song*[7] quotes the lyrics:

> They won't smash up our Pierce Arrow,
> We ain't got none
> They've cut my wages
> But my income tax will be so much smaller
> When I'm paid off,
> I'll be laid off
> Ain't we got fun?

The sheet music published in 1921 by Jerome K. Remick and Co. leaves this chorus out completely,[13] whereas a recording for Edison Records by Billy Jones keeps the reference to the Pierce Arrow, but then continues as in the sheet music: "There's nothing surer / The rich get rich and the poor get laid off / In the meantime,/ In between time/ Ain't we got fun?"[14]

### Reception and performance history

It premièred in the show *Satires of 1920*, where it was sung by Arthur West, then entered the vaudeville repertoire of Ruth Roye.[15] A hit recording by Van and Schenck increased its popularity,[15] and grew into a popular standard.

The song appears in the F. Scott Fitzgerald novel *The Great Gatsby*, when Daisy Buchanan and Gatsby meet again after many years,[10][16] and appears in Dorothy Parker's 1929 short story, "Big Blonde".[17] Warner Brothers used the song in two musicals during the early 1950s: The Gus Kahn biopic *I'll See You in My Dreams* and *The Eddie Cantor Story*.[15] Woody Allen used the song in his 1983 film *Zelig*.[18]

### Notable Recordings

Doris Day for her album "By the Light of the Silvery Moon
The song was featured in the film "By the Light of the Silvery Moon" (1953), and performed by Doris Day and Gordon MacRae.

## 8. Skálafüggetlen hálózat

Egy komplex hálózat **skálafüggetlen**, ha benne a fokszámeloszlás hatványfüggvényt (Yule–Simon eloszlást) követ:

$$\mathbf{P(k) \sim k^{-\gamma}},$$

Az ilyen hálózatoknak viszonylag sok nagy fokszámú csomópontjuk van, és a csomópontok fokszámeloszlása méretfüggetlen (formálisabban ha x egy véletlenül választott csúcs fokszáma, akkor a P(x>n) és P(x>n|x>m) eloszlások csak egy konstans szorzóban különböznek).

Egy másik, nem egyenértékű definíció az, hogyha az összekötött csúcsok fokszámainak szorzatát összegzem, nagy értéket kapok, azaz ha *E* a gráf éleinek halmaza, akkor

$$\sum_{(i,j)\in E} d_i d_j$$

az azonos fokszámeloszlású gráfok között közel maximális. Ez azt jelenti, hogy a nagy fokszámú csúcsok jellemzően össze vannak kötve egymással.

### Kialakulás

Skálafüggetlen gráfok kialakulásának egyik lehetséges módja a preferenciális kapcsolódás. Ez azt jelenti, hogy egy növekvő gráfban annak a valószínűsége, hogy az új csúcs csatlakozzon egy régivel, arányos a régi csúcs fokszámával. Az ilyen módon nyerhető hálózatok csak egy alosztályát alkotják a skálafüggetlen hálózatoknak.

### Tulajdonságok

A skálafüggetlen gráfok kis-világ tulajdonságúak. A klaszterezettség egy hatványfüggvény fordítottja szerint arányos bennük a fokszámmal (vagyis a kis fokszámú pontok sűrű részhálókat alkotnak, és ezeket a részhálókat nagy fokszámú csomópontok kapcsolják össze).

A skálafüggetlen rendszerek rendkívül hibatűrőek a véletlen hibákkal szemben, azaz rendkívül sok véletlenül választott pontot eltávolítható úgy, hogy a rendszer továbbra is összefüggő marad. Másfelől viszont nagyon sérülékenyek a célzott támadásokkal szemben, viszonylag kevés csúcsot eltávolítva a legnagyobb csomópontok közül a hálózat azonban darabjaira hull.

### Alkalmazás

A skálafüggetlenség jelentőségét az adja, hogy számos, fontos gyakorlati szerepet játszó hálózatról kimutatták, hogy ilyen tulajdonságú; például a különféle szociális hálókról, az Internetről, a World Wide Webről, az idegsejtek alkotta hálózatokról, a járványok terjedési útvonalairól vagy a sejtek reakcióútjai.

### Kritika

A skálafüggetlen hálózatok elméletével szembeni egyik kritika, hogy összemosódnak a matematikai modellekből levezetett, és a valódi skálafüggetlen (vagy annak vélt) hálózatokban tapasztalt jellemzők; az elmélet használói gyakran nem teszik egyértelművé, hogy mely tulajdonságokat tekintenek a skálafüggetlenség definíciója részének, mik azok, amik következnek (vagy nagy valószínűséggel következnek) a definícióból, és mik azok, amik esetlegesek.

### Története

A skálafüggetlenség és a preferenciális kapcsolódás felfedezése Barabási Albert-László és munkatársai nevéhez fűződik, akik 1999-ben a Notre Dame Egyetemen a honlapok közötti linkek hálózatát tanulmányozták. Munkatársával Albert Rékával kidolgoztak egy olyan modellt, amely skálafüggetlen hálózatot hoz létre, az Barabási–Albert-modellt.

### Irodalom

- Barabási, Albert-László and Albert, Réka. "Emergence of scaling in random networks". *Science*, 286:509-512, October 15, 1999.
- Lun Li, David Alderson, Reiko Tanaka, John C. Doyle, Walter Willinger: Towards a Theory of Scale-Free Graphs: Definition, Properties, and Implications, 2005
- Mark Newman, Albert-László Barabási, Duncan J. Watts: *The Structure and Dynamics of Networks, Princeton University Press*, 2006 (Benne: Error end attack tolerance of complex networks)

## 9. Kis-világ tulajdonság

Egy **kis-világ tulajdonságú** gráfban vagy hálózatban a csúcsok közötti átlagos távolság a csúcsok számához képest kicsi. Az elnevezés Stanley Milgram kis-világ kísérletéből származik, ami azt vizsgálta, legkevesebb hány személyes ismeretségi kapcsolaton keresztül eljutni egy embertől egy másikig, vagyis mekkora az ismeretségi kapcsolatokat leíró szociális hálóban az átlagos távolság.

A kis-világ tulajdonság számos fontos hálózatra jellemző, például a szociális hálókra, az Internetre vagy a gén-expressziós hálózatokra.

A véletlen gráfok legtöbb fajtája kis-világ tulajdonságú: ha egy nagy átmérőjű gráfba felveszünk néhány véletlen élt, az átmérő nagyon gyorsan csökken. Három gyakran használt, kis-világ tulajdonságú modell az Erdős–Rényi modell, a Watts–Strogatz modell és a Barabási–Albert modell; az átlagos úthossz mindháromban kicsi, de egyéb fontos jellemzőikben, például a klaszterezettségben vagy a fokszámeloszlásban eltérnek.

## 10. Átlagos távolság

Az **átlagos távolság** vagy **átlagos úthossz** a gráfelméletben a pontpárok közötti legrövidebb úthosszak átlaga. A fokszámeloszlás és a klaszterezettség mellett az egyik legfontosabb mérőszám a hálózati topológiában. Az átlagos úthossz mutatja, hogy mennyire hatékony egy hálózat, például hány csomóponton kell áthaladnia egy üzenetnek, vagy mennyi veszteséggel képes áramot közvetíteni egy elektromos hálózat.

Nem összekeverendő az átmérővel, ami a pontpárok közötti legrövidebb úthosszak maximuma.

Számos, a gyakorlatban előforduló hálózatnál, mint például az internet, vagy az ismeretségi hálózatok, az átlagos úthossz viszonylag kicsi, a csúcsok számának logaritmusával arányos. Ez a kis átlagos úthossz a kis-világ tulajdonság egyik feltétele.

## 11. Fokszámeloszlás

A **fokszámeloszlás** a gráfelméletben azt adja meg, hogy a különféle fokszámú csúcsok milyen gyakorisággal fordulnak elő egy gráfban. Erdős Pál és Rényi Alfréd vezette be az 1950-es években a véletlen gráfok vizsgálatára. Az átlagos úthossz és a klaszterezettség mellett az egyik legfontosabb jellemző a hálózati topológiában.

Formálisan a V csúcshalmazú gráf fokszámeloszlása

$$p(k) = \sum_{v \in V \mid \deg(v)=k} 1$$

, illetve a kumulatív fokszámeloszlása

$$P(k) = \sum_{k'=k}^{\infty} p(k')$$

.

P(k) tehát a gráf fokszámának eloszlásfüggvénye egy véletlenül választott csúcsra, p(k) pedig a hozzátartozó sűrűségfüggvény.

A különböző típusú hálózatoknak különböző jellegzetes fokszámeloszlása van, például a skálafüggetlen hálózatoknak hatványfüggvényt közelítő, azaz $P(k) \sim k^{-\gamma}$.

### Irodalom


- Erdos, P. and Renyi, A., 1959, Publ. Math. (Debrecen) **6**, 290.


## 12. Klaszterezettség

A **klaszterezettség** a gráfelméletben azt mutatja meg, hogy mennyire gyakori, hogy egy gráf egy csúcsának szomszédai egymásnak is a szomszédai, azaz milyen közel vannak a csúcsok szomszédai által feszített részgráfok a teljes gráfhoz. A fogalmat Duncan J. Watts és Steven Strogatz vezette be 1998-ban a kis-világ tulajdonság vizsgálatára. A hálózati topológia vizsgálatában az átlagos távolság és a fokszámeloszlás mellett az egyik legfontosabb jellemző.

### Definíció

Irányítatlan gráfban egy csúcs klaszterezettsége annak az aránya, hogy hány él van a szomszédai között, és hogy maximálisan hány lehetne, azaz egy *G*(*V*,*E*) gráfban – a $v_i$ csúcs szomszédainak halmazát $N_i = \{v_j\} : (v_i, v_j) \in E$-vel jelölve – a $v_i$ csúcs klaszterezettsége

$$C_i = \frac{|\{(v_j, v_k) \in E | v_j, v_k \in N_i\}|}{|\{(v_j, v_k) | v_j, v_k \in N_i\}|}$$

Irányított gráfokra a klaszterezettség hasonlóan definiálható, csak az éleket mindkét irányban számolni kell.

Egy másik szokásos megfogalmazásban, jelölje $\lambda_G(v)$ a *v* csúcsot tartalmazó háromszögek számát (azaz azon három csúcsot és három élt tartalmazó részgráfokét, amelyeknek *v* az egyik csúcsa), és $\tau_G(v)$ azoknak a tripleteknek (vagyis két szomszédos élből álló, nem feltétlenül feszített részgráfoknak) a számát, amiknek *v* a középpontja. Ekkor

$$C_i = \frac{\lambda_G(v)}{\tau_G(v)}$$

A teljes gráf klaszterezettsége az egyes csúcsok klaszterezettségének az átlaga:

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i$$

Egy gráf akkor kis-világ tulajdonságú, ha a klaszterezettsége lényegesen nagyobb egy azonos csúcsszámú véletlen gráf klaszterezettségénél, és az átlagos legrövidebb úthossza kicsi.

## Irodalom

D. J. Watts and Steven Strogatz (1998. June). „Collective dynamics of 'small-world' networks”. Nature 393: 440–442.

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